

Phantom inflation with a steplike potential

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The phantom inflation predicts a slightly blue spectrum of tensor perturbation, which might be tested in coming observations. In normal inflation models, the introduction of step in its potential generally results in an oscillation in the primordial power spectrum of curvature perturbation. We will check whether there is the similar case in the phantom inflation with steplike potential. We find that for same potentials, the oscillation of the spectrum of phantom inflation is nearly same with that of normal inflation, the difference between them is the tilt of power spectrum.

I. INTRODUCTION

The results of recent observations are consistent with an adiabatic and nearly scale invariant spectrum of primordial perturbations, as predicted by the simplest models of inflation. The inflation is supposed to have taken place at the earlier moments of the universe [1],[2],[3], which superluminally stretched a tiny patch to become our observable universe today. During the inflation the quantum fluctuations in the horizon will be able to leave the horizon and become the primordial perturbations responsible for the structure formation of observable universe. In this sense, exploring different inflation models is still an interesting issue.

Recently, the phantom field, for which the parameter of state equation $\omega < -1$ and the weak energy condition is violated, has acquired increasingly attention [4], inspired by the wide use of such fields to describe dark energy, e.g. [5],[6],[7],[8], [9],[10],[11]. The simplest realization of phantom field is a normal scalar field with reverse sign in its dynamical term. The quantum theory of such a field may suffer from the causality and stability problems [12, 13]. However, this does not mean that the phantom field is unacceptable. Actions with phantomlike form may be arise in supergravity [14], scalar tensor gravity [15], higher derivative gravity [16], braneworld [17], stringy [18], and other scenarios [19, 20]. The phantom inflation, which is driven by the phantom field, has been proposed [21], and widely studied in [22],[23],[24],[25],[26],[27],[28]. In phantom inflation, the power spectrum of curvature perturbation can be nearly scale invariant. The duality of primordial spectrum to that of normal field cosmology has been studied in [22, 29]. In the meantime the spectrum of tensor perturbation is slightly blue tilt, which is distinguished from that of normal inflationary models [21].

The power spectrum of perturbations in the normal inflation depends on the inflaton potential. In some inflationary models, there may be inflaton potentials with a large number of steps. The steplike

change in the potential will result in a universal oscillation in the spectrum of primordial perturbations [30],[31],[32],[33],[34],[35],[36],[37], and also [38],[39]. A burst of oscillations in the primordial spectrum seems to provide a better fit to the CMB angular power spectrum [40],[41]. In this Letter, we will check whether there is a similar feature in the phantom inflation with steplike potential. We will consider a quadratic potential with a step and a hybrid potential with a step, respectively, and numerically calculate the corresponding power spectrum, and then compare them with that of normal inflationary model.

The plan of this work is as follows. In section II we present the phantom inflation scenario and give the simple analysis of the background evolution with steplike potential. In section III we discuss the calculation of the power spectrum and present numerical results for the primordial spectrum. Section IV contains discussion and conclusions. Note that we work in units such that $\hbar = c = 8\pi G = 1$.

II. THE BACKGROUND OF PHANTOM INFLATION WITH A STEP

The simplest realization of phantom field is a normal scalar field with reverse sign in its dynamical term. This reverse sign results in that, different from the evolution of normal scalar field during the normal inflation, the phantom field during the phantom inflation will be driven by its potential up along its potential, e.g.[9, 10]. Thus if initially the phantom field is in the bottom of potential, analogous to the slow rolling regime of the normal scalar field, the phantom field will upclimbe and enter slow climbing regime. Hereafter, the phantom inflation begins, and after some time the phantom inflation ends and the universe enters into a period dominated by the radiation. The exit from the phantom inflation can be implemented by introducing an additional normal scalar field [21], in which the exit mechanism is similar to the case of hybrid inflation [42, 43], the imposition of back-reaction [26], the wormhole [23], the brane/flux annihilation in string theory [27], or the nonminimally coupling of the phantom to gravity [28].

When the phantom field is minimally coupled to the gravitational field, the Friedmann equation can be writ-

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ten as

$$3H^2 = -\frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (1)$$

where H^2 is positive, which means in all cases for the phantom evolution its dynamical energy must be smaller than its potential energy, thus the phantom inflation is not generally interrupted by the step in despite of the height of step. This can be compared with that in normal inflation, in which it is possible that $\dot{\phi}^2 > V(\phi)$ for a high step and the inflation is interrupted for a short interval. The phantom field satisfies the equation $\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0$. The phantom field is driven to up-climb along its potential is reflected in the minus before V' term. Define the slow-climb parameters [21]

$$\epsilon_{pha} \equiv -\frac{\dot{H}}{H^2}, \quad \delta_{pha} \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} \quad (2)$$

when the conditions $|\epsilon_{pha}| \ll 1$ and $|\delta_{pha}| \ll 1$ are satisfied, Eq (1) can be solved semianalytically. Then we have $a \sim e^{Ht}$ approximately, which means the universe enters into the inflationary phase driven by the phantom field.

The numerical solutions of evolution equations are required for accurately evaluating the perturbation spectrum. We shift the independent variable to $\alpha = \ln a$, which will facilitate the numerical integration. With this replacement and using the energy conservation equation, we have

$$H_\alpha = \frac{1}{2}H\phi_\alpha^2, \quad (3)$$

$$\phi_{\alpha\alpha} + \left(\frac{H_\alpha}{H} + 3\right)\phi_\alpha - \frac{1}{H^2}V' = 0. \quad (4)$$

where the subscript α denotes differentiation for α and the prime denotes differentiation with the scalar field ϕ .

In general, the step in the potential can be modelled by introducing the term proportional to $\tanh(\frac{\phi - \phi_{step}}{\delta})$. We consider a simple inflaton potential $m^2\phi^2$, then this potential with a step can be given by

$$V(\phi) = \frac{1}{2}m^2\phi^2 \left(1 + \beta \tanh\left(\frac{\phi - \phi_{step}}{\delta}\right)\right). \quad (5)$$

This potential has a step at $\phi = \phi_{step}$ with size and gradient governed by β and δ . We will focus on small features in the potential, and thus will limit the parameter β small. In this case, the phantom will upclimb continuously through the step while the inflation will not be ceased.

The numerical results with the potential (5) can be seen in Figure 1. We can see that H is nearly constant in the inflationary era, but there are some differences from the normal background. H increases slowly in the phantom inflation compared with decrease slowly in the

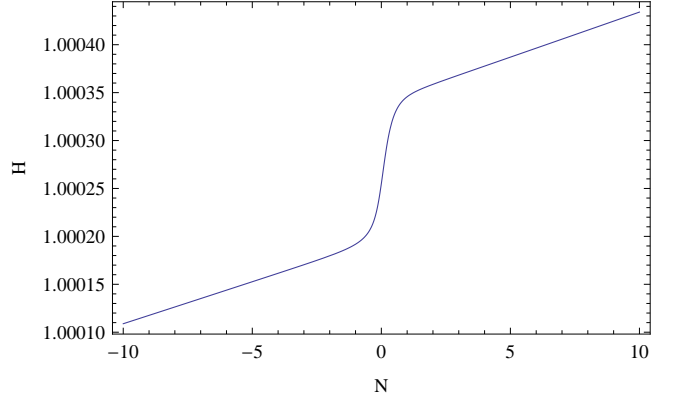


FIG. 1: Evolution of the Hubble parameter H . H is in the unit of H_0 which is a model dependent parameter. H increases and is different from normal inflation

normal inflation, since in the phantom inflation the energy density is increased. There is a very abrupt change due to the existence of the step.

The slow-climb parameters in term of α can be changed as follows

$$\epsilon_{pha} = -\frac{H_\alpha}{H}, \quad \delta_{pha} = -\frac{\phi_{\alpha\alpha}}{\phi_\alpha} - \frac{H_\alpha}{H}. \quad (6)$$

The introduction of the step leads to a deviation from slow-climb inflation, though during this interval it is still inflation. We have plotted the evolution of the two slow-climb parameters ϵ_{pha} and δ_{pha} around the time when the field crosses the step in Figure 2.

III. THE PERTURBATION OF PHANTOM INFLATION WITH A STEP

We will investigate the power spectrum of curvature perturbations during the phantom inflation. In the scalar case it is advantageous to define a gauge invariant potential [44],[45]

$$u = -z\mathcal{R} \quad (7)$$

where $z \equiv a\sqrt{2|\epsilon_{pha}|}$, and a denotes the scale factor, H is the Hubble parameter, and the dot is the derivative with respect to the physical time t .

The equation of motion u_k in the momentum space is

$$u_k'' + \left(k^2 - \frac{z''}{z}\right)u_k = 0, \quad (8)$$

where the prime denotes differentiation with respect to conformal time and k is the wave number. The solution depends on the relative sizes of k^2 and z''/z . The z''/z term can be expressed as $2a^2H^2$ plus terms that are small during the phantom inflation. In general, on subhorizon scales $k^2 \gg z''/z$, the solution of perturbation is a plane wave

$$u_k \rightarrow \frac{1}{\sqrt{2k}} e^{-ik\tau}. \quad (9)$$

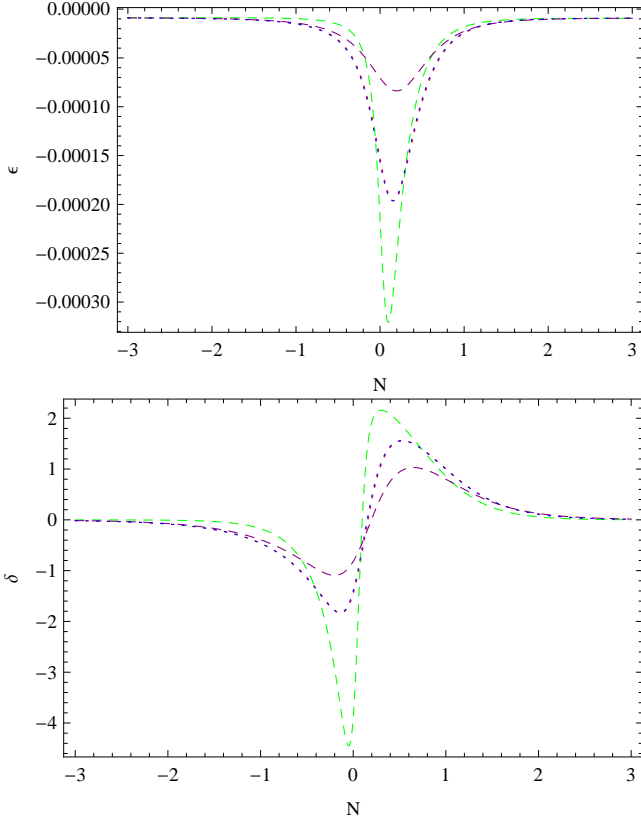


FIG. 2: Evolution of the two slow-climb parameters ϵ_{pha} (top) and δ_{pha} (bottom) with the introduction of the step. Different of the β and δ parameters: $\beta = 10^{-2}, \delta = 10^{-3}$ (red dotted line), $\beta = 10^{-2}, \delta = 5 \times 10^{-4}$ (green dashed line), $\beta = 5 \times 10^{-3}, \delta = 10^{-3}$ (purple dashed line), $\beta = 5 \times 10^{-3}, \delta = 5 \times 10^{-4}$ (blue dotted line).

The $\frac{1}{\sqrt{2k}}$ is obtained by the quantization of mode function u_k . Here a normal quantization condition has actually been applied, like that in normal field, which seems inconsequential for phantom field. However, it is generally thought that the phantom field might be only the approximative simulation of a fundamental theory below certain physical cutoff, and the full theory should be well quantized. In another viewpoint, we might assume that initially there is not phantom field, thus the perturbation deep inside the horizon follows normal quantization condition, then the evolution with $w < 1$ emerges for a period, which is simulated phenomenologically by the phantom field, as in island cosmology [46] or [27]. Thus the primordial perturbation induced by the phantom fields has to have a normal quantization condition as its initial condition, or it cannot be matched to that of initial background. While on superhorizon scales $k^2 \ll z''/z$ the dominated mode is

$$u_k \propto z \quad (10)$$

which means that the curvature perturbation

$$|\mathcal{R}_k| = |u_k/z|, \quad (11)$$

is constant.

The spectrum $\mathcal{P}_{\mathcal{R}}(k)$ is defined as

$$\langle \mathcal{R}_{k_1} \mathcal{R}_{k_2}^* \rangle = \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}} \delta^3(k_1 - k_2), \quad (12)$$

and is given by

$$\mathcal{P}_{\mathcal{R}}^{1/2}(k) = \sqrt{\frac{k^3}{2\pi^2}} \left| \frac{u_k}{z} \right|. \quad (13)$$

We will numerically calculate $\mathcal{P}_{\mathcal{R}}$. However, before this it is interesting to show the result of $\mathcal{P}_{\mathcal{R}}$ in the slow climbing approximation, which is

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{2|\epsilon_{pha}|} \left(\frac{H}{2\pi} \right)^2 \left(\frac{k}{aH} \right)^{n_{\mathcal{R}}-1}, \quad (14)$$

where the spectrum index is $n_{\mathcal{R}} - 1 \simeq -4\epsilon_{pha} + 2\delta_{pha}$. This power spectrum may be either blue or red. The results are dependent on the relative magnitude of ϵ_{pha} and δ_{pha} , see [21] for the details. Eq.(14) is valid only when the slow climb approximation is satisfied, i.e. $|\epsilon_{pha}| \ll 1$ and $|\delta_{pha}| \ll 1$, however, when the potential has a sharp step, the deviation of δ is large, see Fig 2. In this case, we have to evolve the full mode equation numerically without any approximations.

The perturbation equation (8), with the replacement $\alpha = lna$, can be written as

$$u_{\alpha\alpha} + \left(1 + \frac{H_{\alpha}}{H}\right)u_{\alpha} + \left(\frac{k^2}{e^{2\alpha}H^2} - \frac{z''/z}{e^{2\alpha}H^2}\right)u = 0 \quad (15)$$

with

$$\frac{z''}{z} = a^2 H^2 \left(2 - 5\frac{H_{\alpha}}{H} - 2\frac{H_{\alpha}^2}{H^2} - 4\frac{H_{\alpha}}{H} \frac{\phi_{\alpha\alpha}}{\phi_{\alpha}} + \frac{V''}{H^2} \right) \quad (16)$$

The evolution of spectrum is governed by the competition between the k^2 and z''/z terms. The overall normalization of $\mathcal{P}_{\mathcal{R}}$ is proportional to m^2 , ϕ_{step} determines the wavelength at which the feature appears. The dominant contribution to z''/z is from the V'' term and is proportional to β/δ^2 . Thus the range of k affected by the feature roughly depends on the square root of β/δ^2 .

We plot the z''/z term in Fig 3 and solve the equation numerically and give the results of power spectrum in Fig 4. In Fig 3, we can see that the z''/z term is very different from $2a^2H^2$. It has a sharp oscillation near the step. Fig 4 shows the power spectrum of phantom inflation with potential of Eq.(5). We plot it with three group parameters and can see that the introduction of step results in a large deviation from slow climbing inflation and then a burst of oscillations superimposed on the nearly scale invariant scalar power spectrum. The magnitude and extent of oscillation is dependent on the amplitude and gradient of the step. Thus as in normal inflation, these oscillations will inevitably leave interesting imprints in the CMB angular power spectrum, which might provide a better fit e.g.[40],[41].

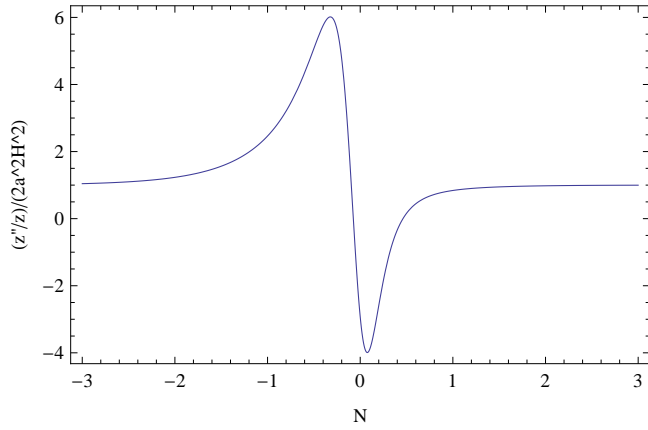


FIG. 3: Evolution of z''/z for $\beta = 0.01$ and $\delta = 0.001$ with the e-folding number of inflation N and we have set $N=0$ at the step in the potential

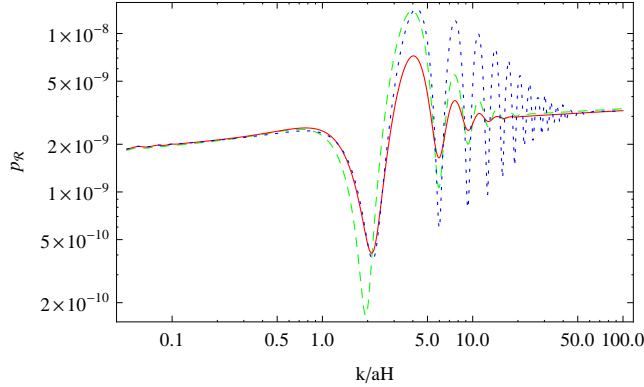


FIG. 4: The power spectrum of curvature perturbation for the phantom inflation with the potential (5), $\beta = 5 \times 10^{-3}$, $\delta = 4 \times 10^{-2}$ (true red line), $\beta = 5 \times 10^{-3}$, $\delta = 10^{-2}$ (dotted blue line), $\beta = 10^{-2}$, $\delta = 4 \times 10^{-2}$ (dashed green line).

We compare the primordial spectrum of phantom inflation and normal inflation in Fig 5, the top line shows the primordial power spectrum of phantom inflation for the potential with $m = 7.5 \times 10^{-6}$, $\beta = 10^{-2}$, $\delta = 5 \times 10^{-2}$. The normal inflation model with a same potential has a red spectrum $n_{\mathcal{R}} < 1$, however, the phantom inflation background model has a blue spectrum $n_{\mathcal{R}} > 1$. This is because what we consider here is the potential with $m^2\phi^2$. In principle, we can have the models of phantom inflation with $n_{\mathcal{R}} < 1$ by choosing a suitable potential [21]. We can consider a model of phantom inflation with the potential

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 \left(1 + \beta \tanh\left(\frac{\phi - \phi_{step}}{\delta}\right) \right) \quad (17)$$

where V_0 is constant which dominates the potential. This potential is same with that of hybrid inflation with normal field. Fig 6 shows that $n_{\mathcal{R}} < 1$ which is similar to that of the normal inflation with $m^2\phi^2$.

The power spectrum of tensor perturbation is only dependent on ϵ_{pha} . We set the parameter β small, thus actually $|\epsilon_{pha}| \ll 1$ around the step. In this case, the tensor spectrum is hardly affected, which remain nearly scale invariant. However, due to $\epsilon_{pha} < 0$, thus $n_T \simeq -2\epsilon_{pha}$ is slightly blue tilt for the phantom inflation [21], which is distinguished from the normal inflation, in which n_T is generally red tilt.

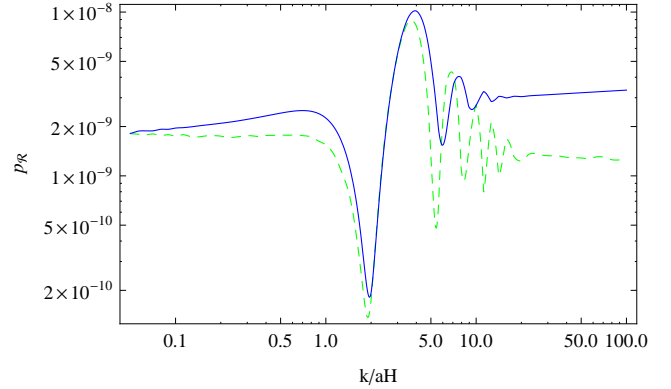


FIG. 5: Effects of a step in the potential on the power spectrum of curvature perturbation for the phantom inflation and the normal inflation, respectively. The blue solid line corresponds to the phantom inflation with $m = 7.5 \times 10^{-6}$, $\beta = 10^{-2}$, $\delta = 5 \times 10^{-2}$ and the green dashed line corresponds to the normal inflation with $m = 7.5 \times 10^{-6}$, $\beta = 10^{-2}$, $\delta = 2 \times 10^{-2}$

IV. CONCLUSION AND DISCUSSION

The phantom field can naturally appear in effective actions of some theories, which might be only the approximative simulation of a fundamental theory below certain physical cutoff. Thus the phantom cosmology have been widely studied. The phantom inflation predicts a slightly blue spectrum of tensor perturbation, which is distinguished from that of the normal inflation, in which the tensor perturbation is generally red tilt. This is a smokegun for the phantom inflation, which might be tested in coming observations.

In normal inflation models, the introduction of step in its potential generally results in an oscillation in the primordial power spectrum of curvature perturbation. In this Letter, we find that for same potentials with the step, the oscillation of the spectrum of phantom inflation can be nearly same with that of normal inflation, and the magnitude and extent of oscillation is dependent on the amplitude and gradient of the step. The difference between them is the tilt of power spectrum. However, the same tilt can be obtained by considering a different potential of the phantom inflation.

In general, $\dot{\phi}^2$ for the phantom must be smaller than its potential energy in all time. Thus the phantom in-

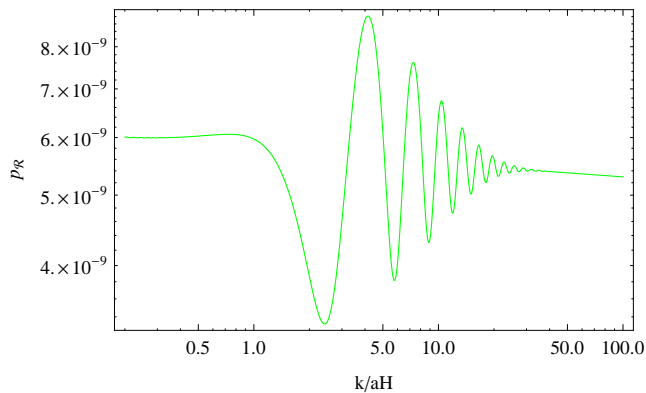


FIG. 6: The power spectrum of curvature perturbation for the phantom inflation with the potential (17), $V_0 = 3.7 \times 10^{-14}$, $m = 3.2 \times 10^{-8}$, $\phi_{step} = 0.0125$, $\beta = 5 \times 10^{-4}$, $\delta = 10^{-5}$. Corresponding to the spectra index $n_s < 1$.

flation is not interrupted by the step in despite of the height of step. This can be compared with that in nor-

mal inflation, in which it is possible that for a high step the inflation is interrupted for a short interval. Here, we have limited the parameter β small, however, it is interesting to consider the phenomena of $\beta \gg 1$, i.e. there is a large step, by which the density of dark energy observed might be linked to that of inflation, as in the eternal expanding cyclic scenario, e.g.[47],[48]. This might lead to a lower CMB quadrupole in observable universe if the step is just in the position of potential, in which the perturbation with Hubble scale leaves the horizon during the phantom inflation, as in the bounce inflation model [38],[39]. It is possible that the phantom inflation with steplike potential can be effectively implemented in certain warped compactifications with the brane/flux annihilation, e.g.[27]. We expect to back to the relevant issues in the coming works.

Acknowledgments

This work is supported in part by NSFC under Grant No:10775180, 11075205, in part by the Scientific Research Fund of GUCAS(NO:055101BM03), in part by National Basic Research Program of China, No:2010CB832804

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